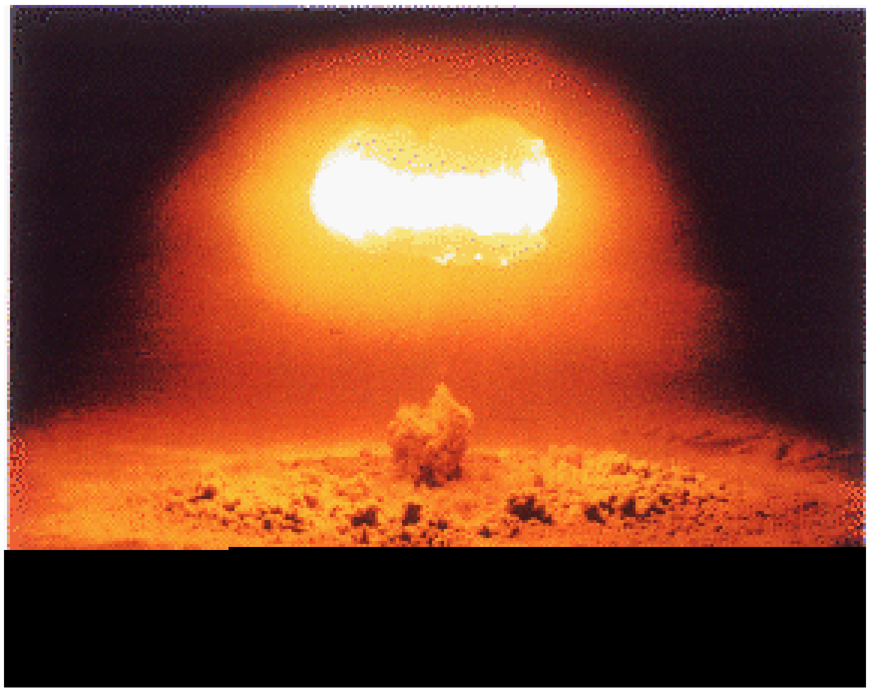
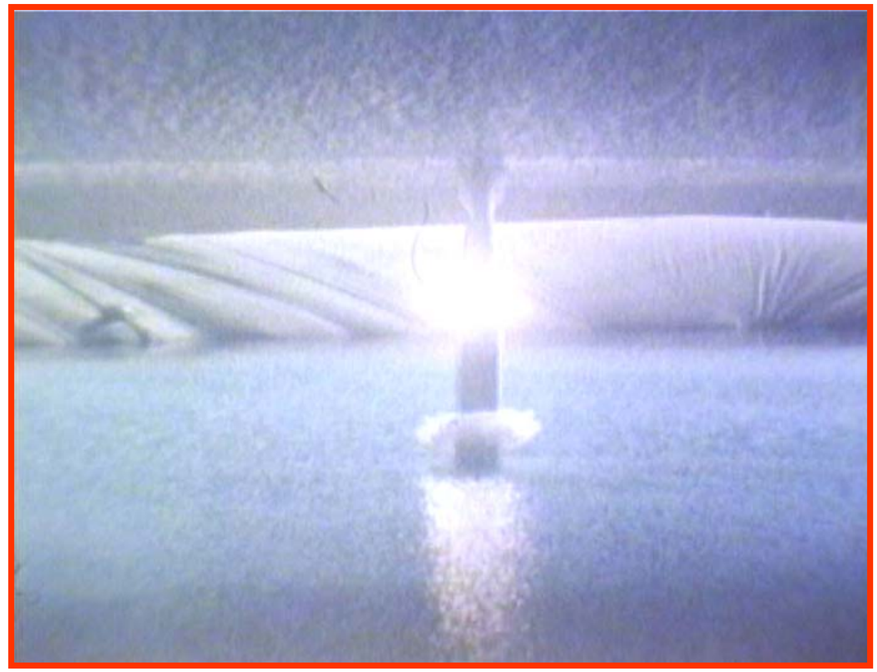
Underwater Bomb Trajectory Prediction

Bomb Strike for Mine Clearance

ONR JDAM Assault Breaching System (JABS)





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SRI Bomb Trajectory Experiment

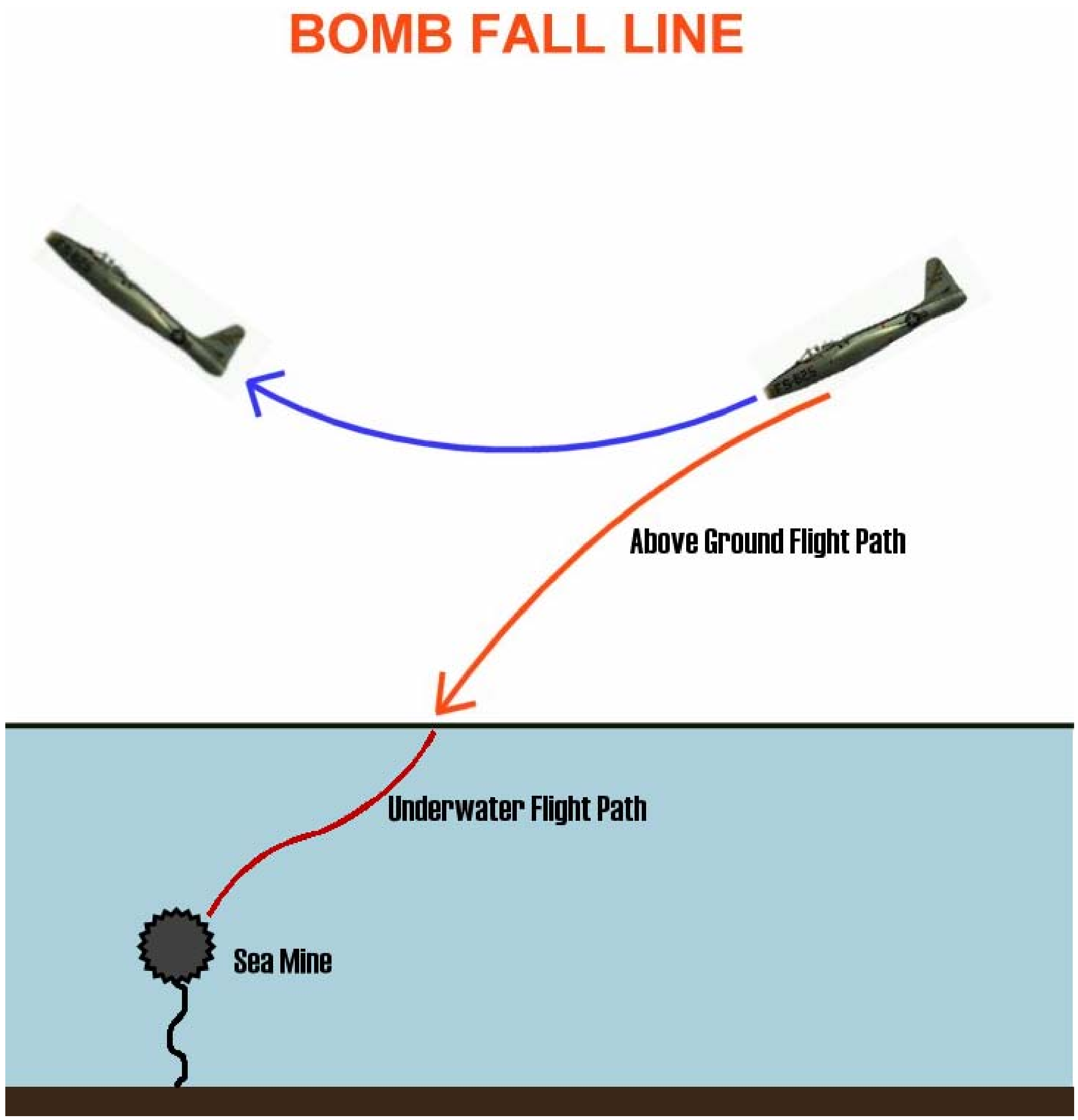
- SRI International performed an experimental research program in which 1/12-scale high fidelity Mk84 bombs were launched into a deep-water pool at velocities of up to about 1000 ft/s.

- Using two underwater high-speed video cameras, they determined the underwater trajectory of the Mk84 bombs for a nominal vertical entry and for different possible tail configurations included a complete warhead section with (1) a tail section and four fins, (2) a tail section and two fins, (3) a tail section and no fins, and (4) no tail section.

SRI Experimental Data (Two-Dimensional, 12 Sets)

- With a Tail Section: → COM location only, no orientation data
  - No Fin: Test-16, -17, -18
  - 2 Fins: Test-10, -11, -19
  - 4 Fins: Test-2, -3, -4.
- With a Tail Section → COM location and Orientation
  - Test-13, -14, -15
  - Only the three sets of data are used for STRIKE35 development and Verification

Prediction of Bomb Maneuvering Trajectory



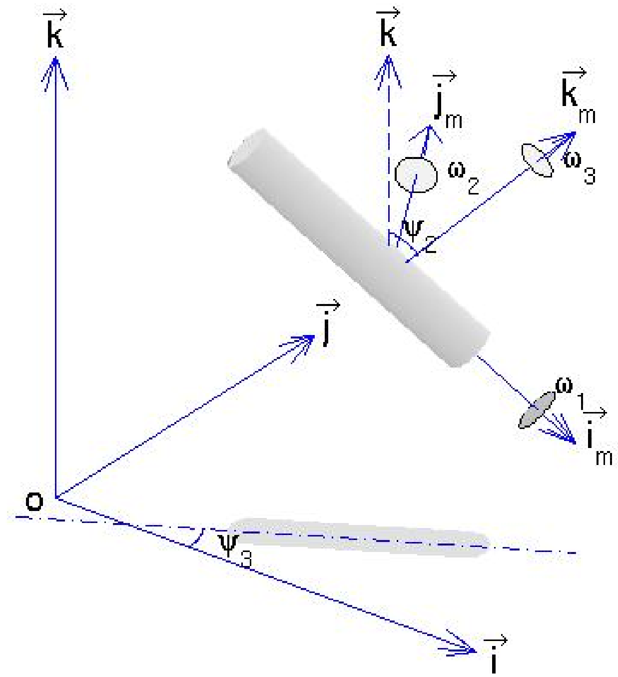
3D Bomb Maneuvering Model (STRIKE35)

- Triple Coordinate Systems
- Momentum Equations
- Moment of Momentum Equations
- Parameterization of Hydrodynamic Forces and Torques on Bomb
  - Supercavitaion
  - Bubble Dynamics

Triple Coordinate Transform

- Earth-fixed coordinate (E-coordinate)
- Bomb’s main-axis following coordinate (M-coordinate)
- Hydrodynamic force following coordinate (F-coordinate).

E and M Coordinate Systems



$$\mathbf{j}_M = \mathbf{k} \times \mathbf{i}_M, \qquad \mathbf{k}_M = \mathbf{i}_M \times \mathbf{j}_M$$

E-Coordinate, $F_E(O, \mathbf{i}, \mathbf{j}, \mathbf{k})$

- COM Position: $\mathbf{X} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$,

- Translation velocity:

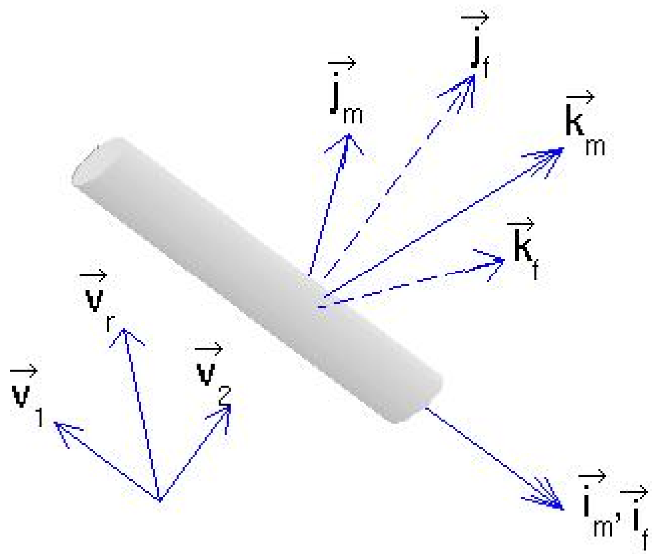
$$d\mathbf{X}/dt = \mathbf{V}, \quad \mathbf{V} = (u, v, w)$$

Transform Between E- and M-Coordinate Systems

$$ {}_M^E\mathbf{R}(\psi_2, \psi_3) \equiv \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} : $$

$$ = \begin{bmatrix} \cos\psi_3 & -\sin\psi_3 & 0 \\ \sin\psi_3 & \cos\psi_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\psi_2 & 0 & \sin\psi_2 \\ 0 & 1 & 0 \\ -\sin\psi_2 & 0 & \cos\psi_2 \end{bmatrix}, $$

F-Coordinate System



E- and F-Coordinate Transform

$$\mathbf{i}_F = \mathbf{i}_M = \begin{bmatrix} r_{11} \\ r_{21} \\ r_{31} \end{bmatrix}, \qquad \mathbf{j}_F = \mathbf{V}_2 / |\mathbf{V}_2|, \qquad \mathbf{k}_F = \mathbf{i}_F \times \mathbf{j}_F.$$

$${}^{E}_{F}\mathbf{R}(\psi_2, \psi_3, \phi_{MF}) \equiv \begin{bmatrix} r_{11} & r'_{12} & r'_{13} \\ r_{21} & r'_{22} & r'_{23} \\ r_{31} & r'_{32} & r'_{33} \end{bmatrix},$$

Momentum Equation in E-Coordinate System

$$\frac{d}{dt}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ g(\rho_w / \rho - 1) \end{bmatrix} + \frac{\rho_w}{\rho}\frac{D\mathbf{V}_w}{Dt} + \frac{1}{\rho\Pi}(\mathbf{F}_h + \mathbf{F}_v),$$

$\mathbf{F}_h$ is hydrodynamic force (drag, lift)

$\mathbf{F}_v$ is the bubble force (drag, lift)

Moment of Momentum Equation in M-Coordinate System

$$\mathbf{J} \cdot \frac{d\boldsymbol{\omega}}{dt} = \mathbf{M}_w + \mathbf{M}_b + \mathbf{M}_h + \mathbf{M}_v,$$

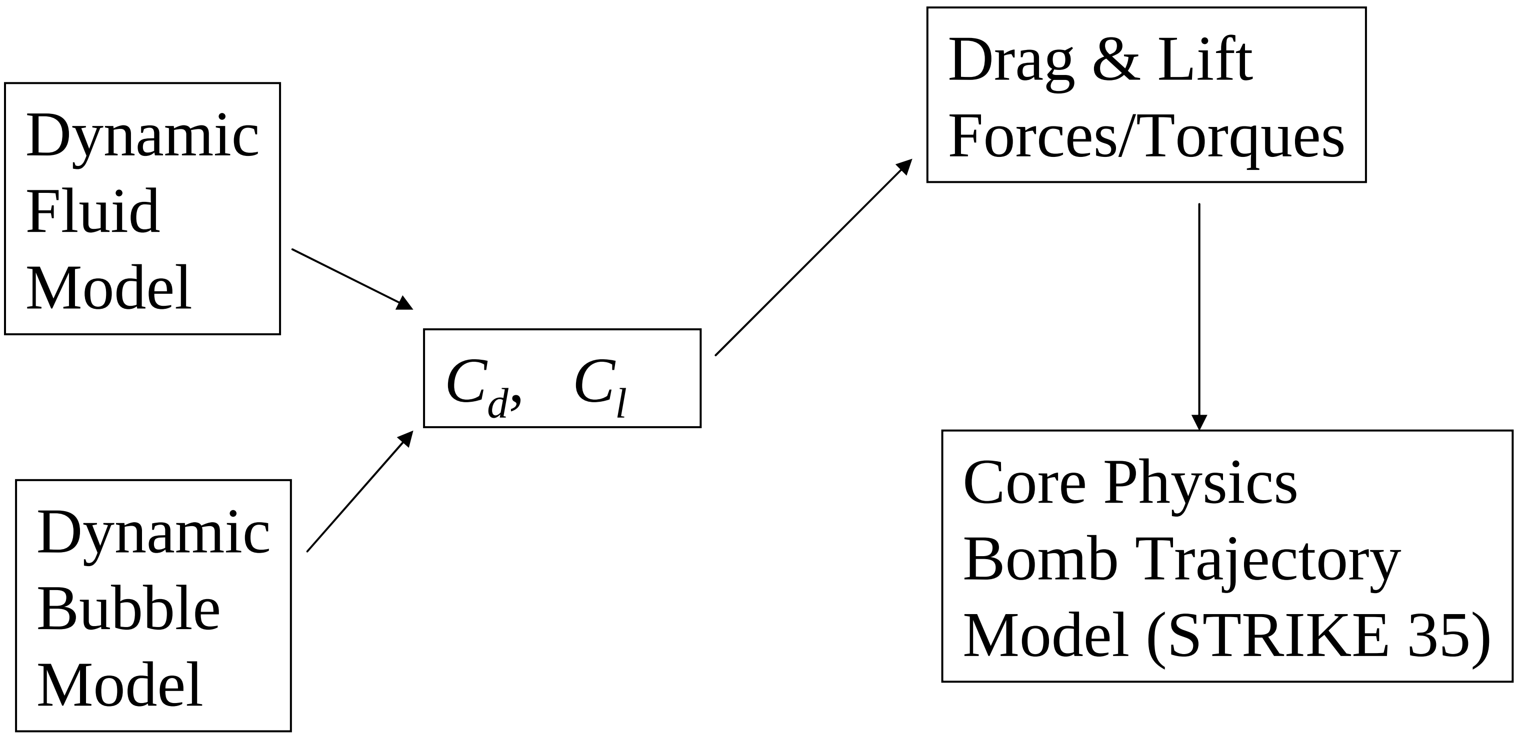
Inertial terms are small

M-Coordinate

The moment of gyration tensor for the axially Symmetric cylinder is a diagonal matrix

$$\mathbf{J} = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix},$$

Bomb Trajectory Modeling



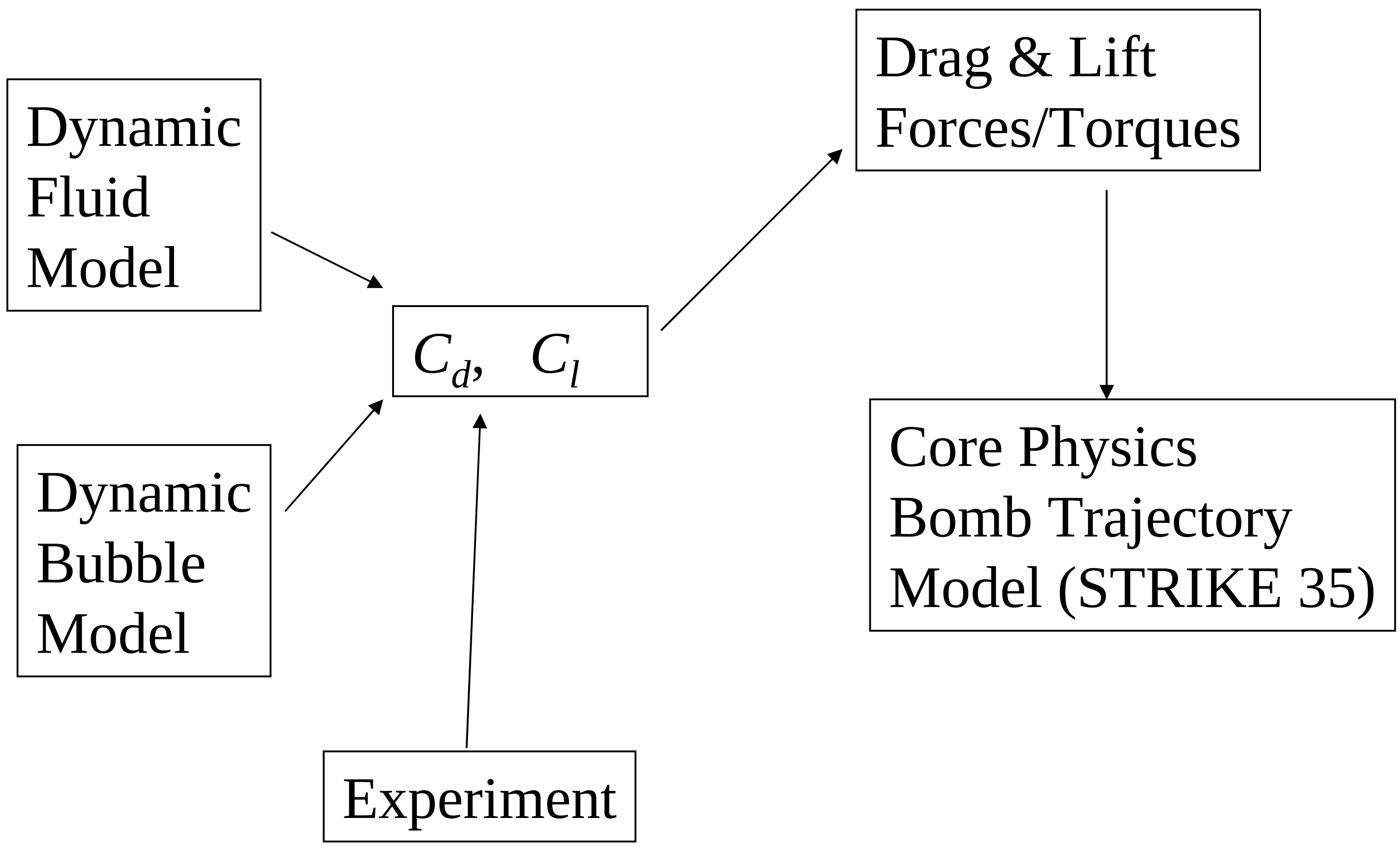
There is no existing formulae for calculating C_d and C_l for MK-84 Bomb.

Two-Step Modeling

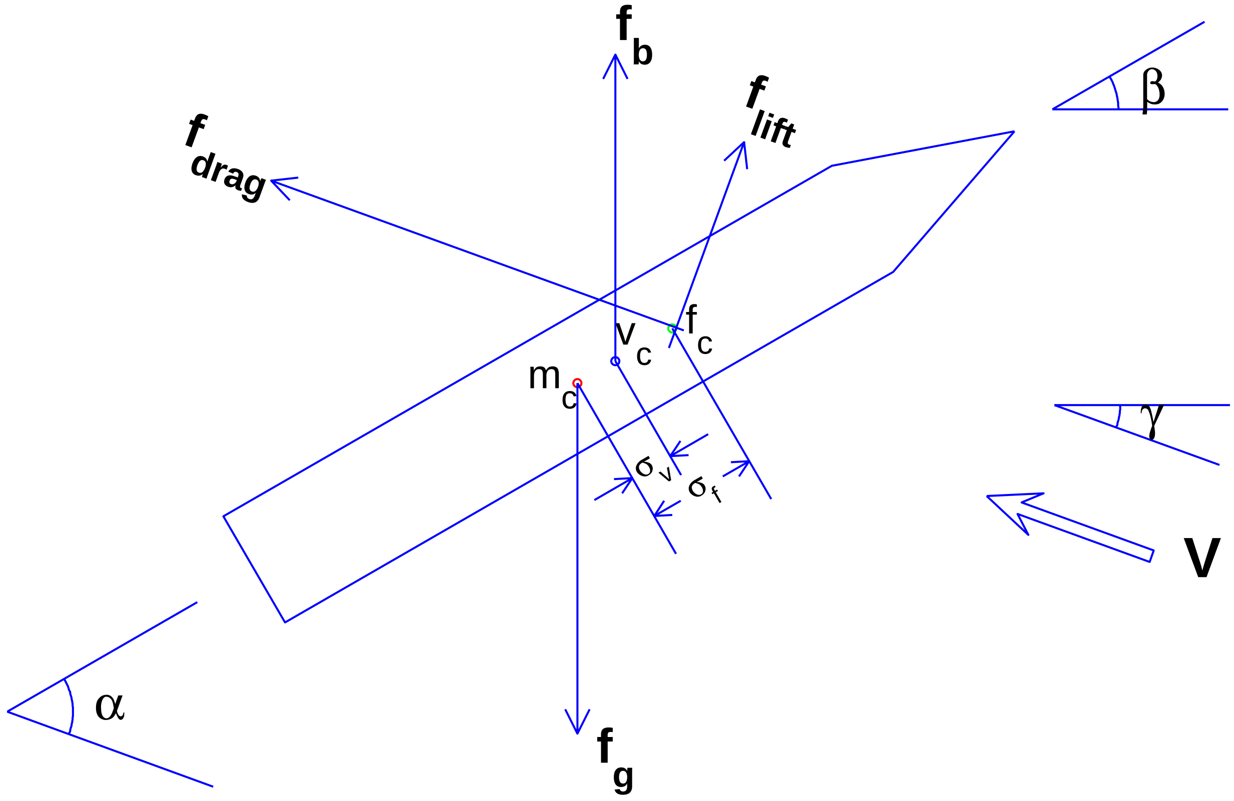
- (1) Determine drag and lift coefficients for a particular bomb (usually from experiments)

- (2) Predict bomb trajectory using stand-alone bomb strike model (STRIKE-35) with the known drag and lift coefficients.

STRIKE 35 Modeling



Dynamical Determination of Drag/Lift Coefficients



β bomb elevation angle

γ : bomb velocity angle

$\alpha = \beta - \gamma$: attack angle

$\mathbf{f}_g$: Gravity

$\mathbf{V}$: water velocity relative to Bomb

m_c : center of mass

v_c : center of volume

f_c : center of drag and lift forces

$\mathbf{f}_{drag}$: drag force

$\mathbf{f}_{lift}$: lift force

$\mathbf{f}_b$: buoyancy force

σ_f : the distance between m_c and f_c

σ_v : the distance between m_c and v_c

Theoretical Base

$$m\frac{d\mathbf{v}}{dt} = \left(\rho\Pi - m\right) g\mathbf{k} + f_{drag}\mathbf{e}_d + f_{lift}\mathbf{e}_l$$

$$\mathbf{I}\bullet\frac{d\boldsymbol{\Omega}}{dt} = \mathbf{r}_v \times \mathbf{f}_b + \mathbf{r}_f \times \left(\mathbf{f}_{drag} + \mathbf{f}_{lift}\right) + \mathbf{M}_r$$

Here, **v** is the translation velocity of COM,

$\boldsymbol{\Omega}$ is the angular velocity.

Determination of C_d and C_L from Experimental Data

$$C_d = \frac{\left(\rho\Pi - m\right) g\mathbf{k} \bullet \mathbf{e}_d - m d\mathbf{v} / dt \bullet \mathbf{e}_d}{\frac{1}{2}\rho D L V^2}$$

$$C_l = \frac{\left(\rho\Pi - m\right) g\mathbf{k} \bullet \mathbf{e}_l - m d\mathbf{v} / dt \cdot \mathbf{e}_l}{\frac{1}{2}\rho D L V^2}$$

Analytical Formulae for (C_d, C_l) Using Three Sets of SRI MK-84 Data without Tail

$$C_d = \begin{cases} 8\sin(2\alpha)\left(\dfrac{\mathrm{Re}_{ref}}{\mathrm{Re}}\right)^2 + 0.02 & \text{if } \sin(2\alpha) \geq 0 \text{ and } \mathrm{Re} \geq \mathrm{Re}_{ref} \\ 0.34\left|\sin(2\alpha)\right|\left(\dfrac{\mathrm{Re}_{ref}}{\mathrm{Re}}\right) + 0.02 & \textit{otherwise} \end{cases}$$

$$C_l = \begin{cases} 2.5\sin(2\alpha)\min\left[\left(\dfrac{\mathrm{Re}}{\mathrm{Re}_{ref}}\right)^{1.2}, \left(\dfrac{\mathrm{Re}_{ref}}{\mathrm{Re}}\right)^{1.2}\right] & \text{if } \sin(2\alpha) \geq 0 \\ 0.16\sin(2\alpha) & \text{if } \sin(2\alpha) < 0 \end{cases}$$

$$\mathrm{Re}_{ref} = 1.51 \times 10^7$$

Determination of Center of Hydrodynamic Force from Experimental Data

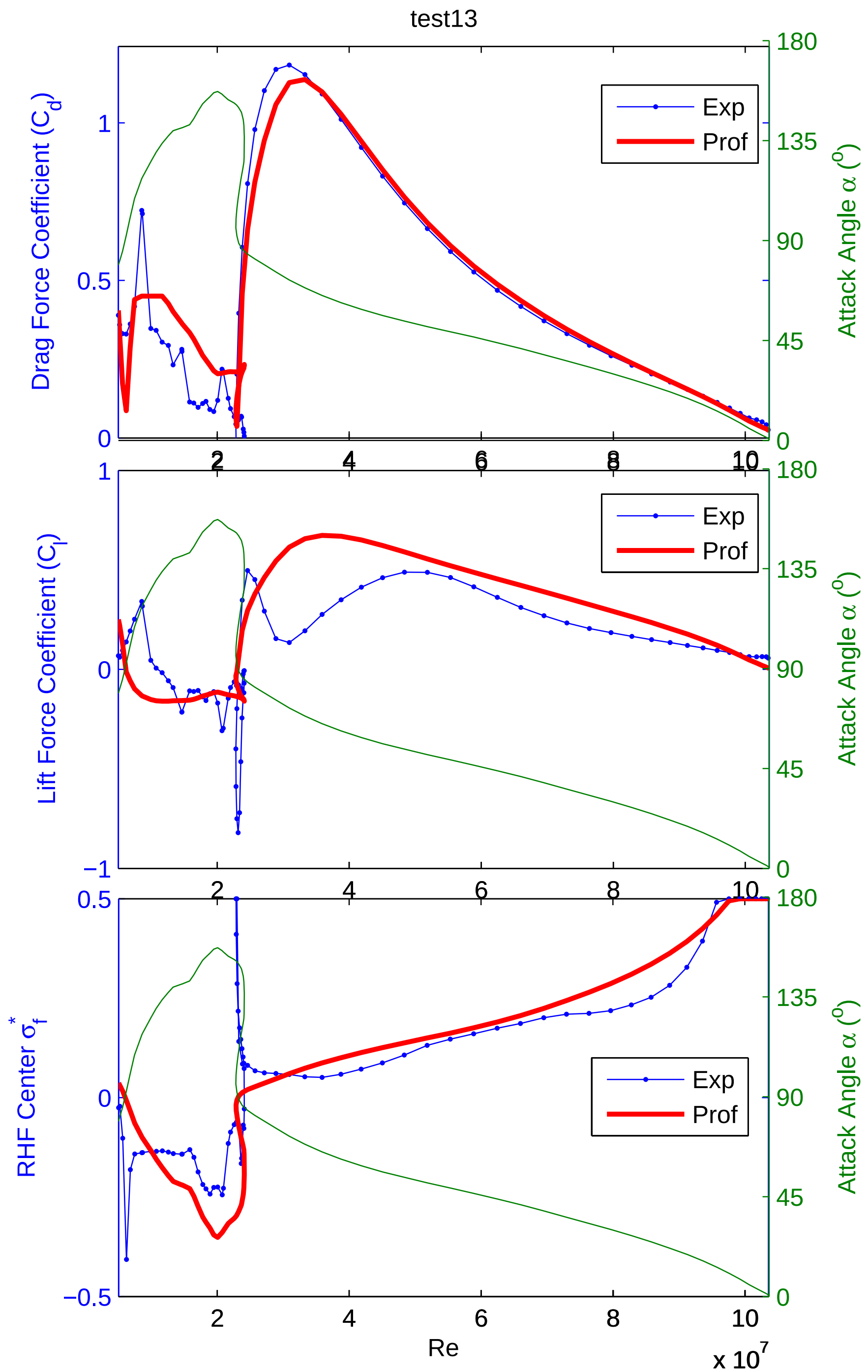
$$\sigma_f^* = \frac{\sigma_f - \sigma_v}{L}$$

$$\sigma_f^* L = \frac{\left[\mathbf{I} \bullet \left(\frac{d\Omega}{dt} \mathbf{e}_r + \Omega \frac{d\mathbf{e}_r}{dt} \right) \bullet \mathbf{e}_r - \sigma_v \left(\mathbf{e}_b \times \mathbf{f}_b \right) \bullet \mathbf{e}_r + \frac{1}{2} C_f \rho D L \left(\frac{L^2 V_r}{6} + \frac{L^2 \left| \Omega \sigma_v \right|}{8} + 2 V_r \sigma_v^2 + \frac{\left| \Omega \sigma_v^3 \right|}{2} \right) \right]_r \Omega}{\mathbf{e}_b \times \left(\frac{1}{2} C_d \rho D L V^2 \mathbf{e}_d + \frac{1}{2} C_l \rho D L V^2 \mathbf{e}_l \right) \bullet \mathbf{e}_r} - \sigma_v$$

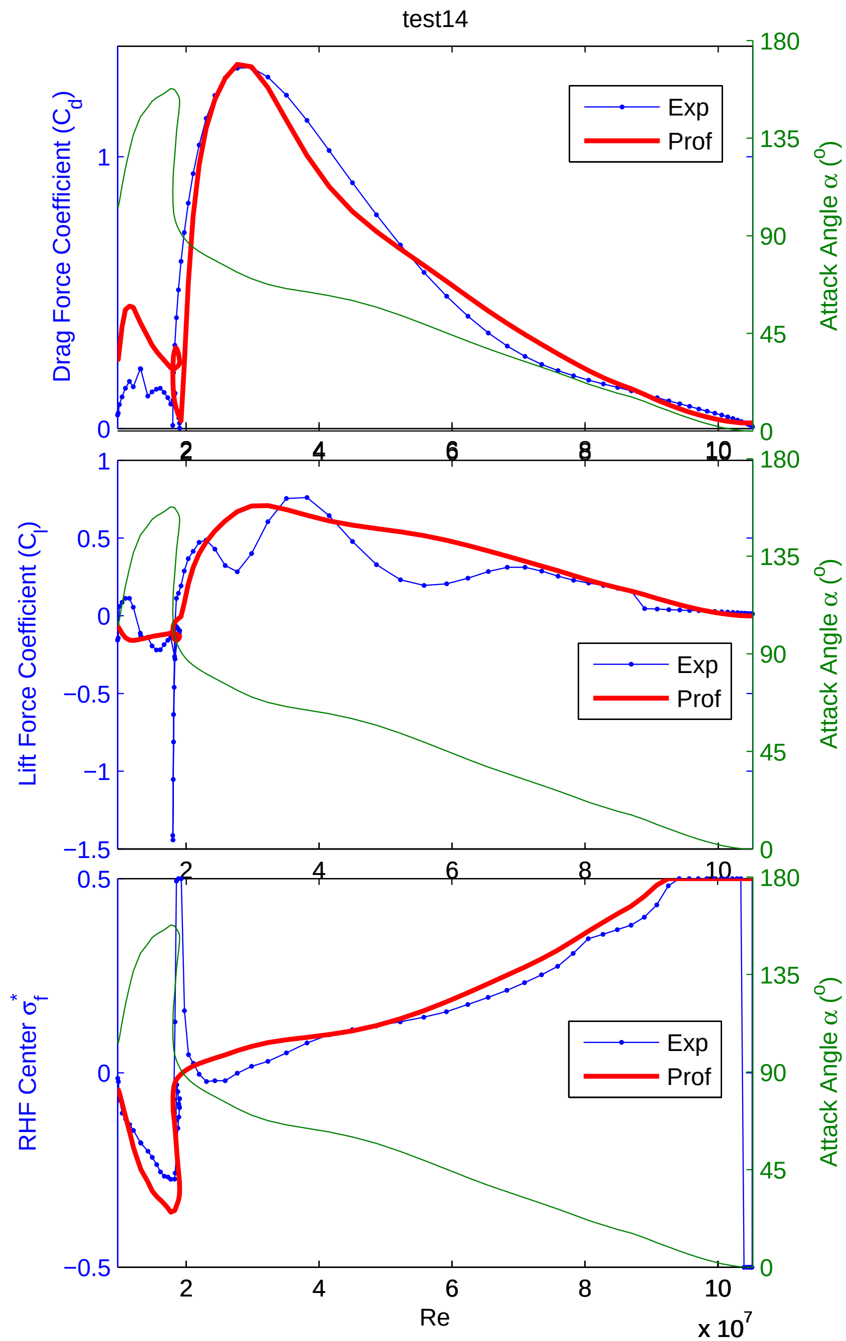
Analytical Formulae for σ_f^* Using Three Sets of SRI MK-84 Data without Tail

$$\sigma_f^* = \frac{\sigma_f - \sigma_v}{L} = \frac{1}{8}\sinh\left(\frac{3}{2}\left(\frac{\pi}{2} - \alpha\right)\right) \quad \text{and} \quad -\frac{1}{2} \le \frac{\sigma_f^*}{L} \le \frac{1}{2}$$

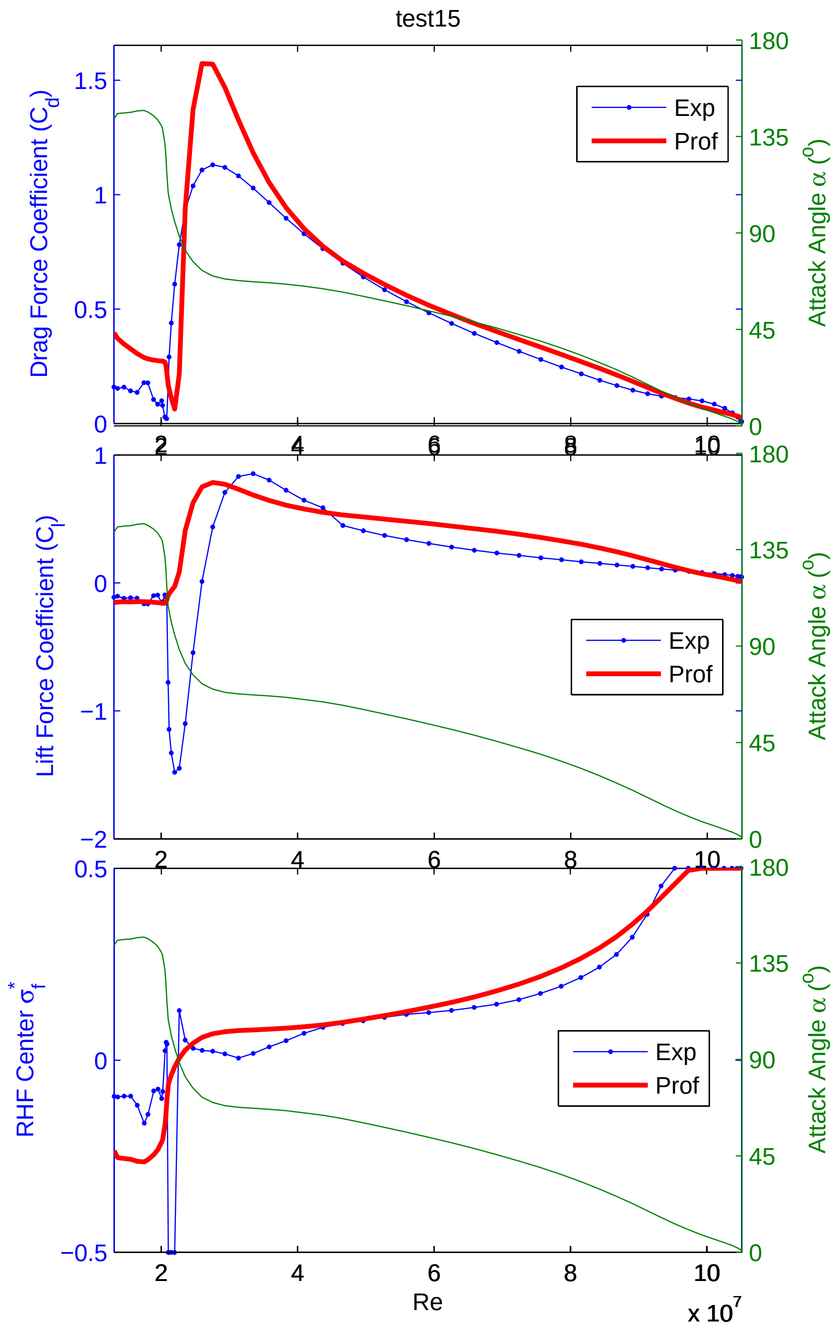
- Test-13
- C_d, C_l, σ_f^*



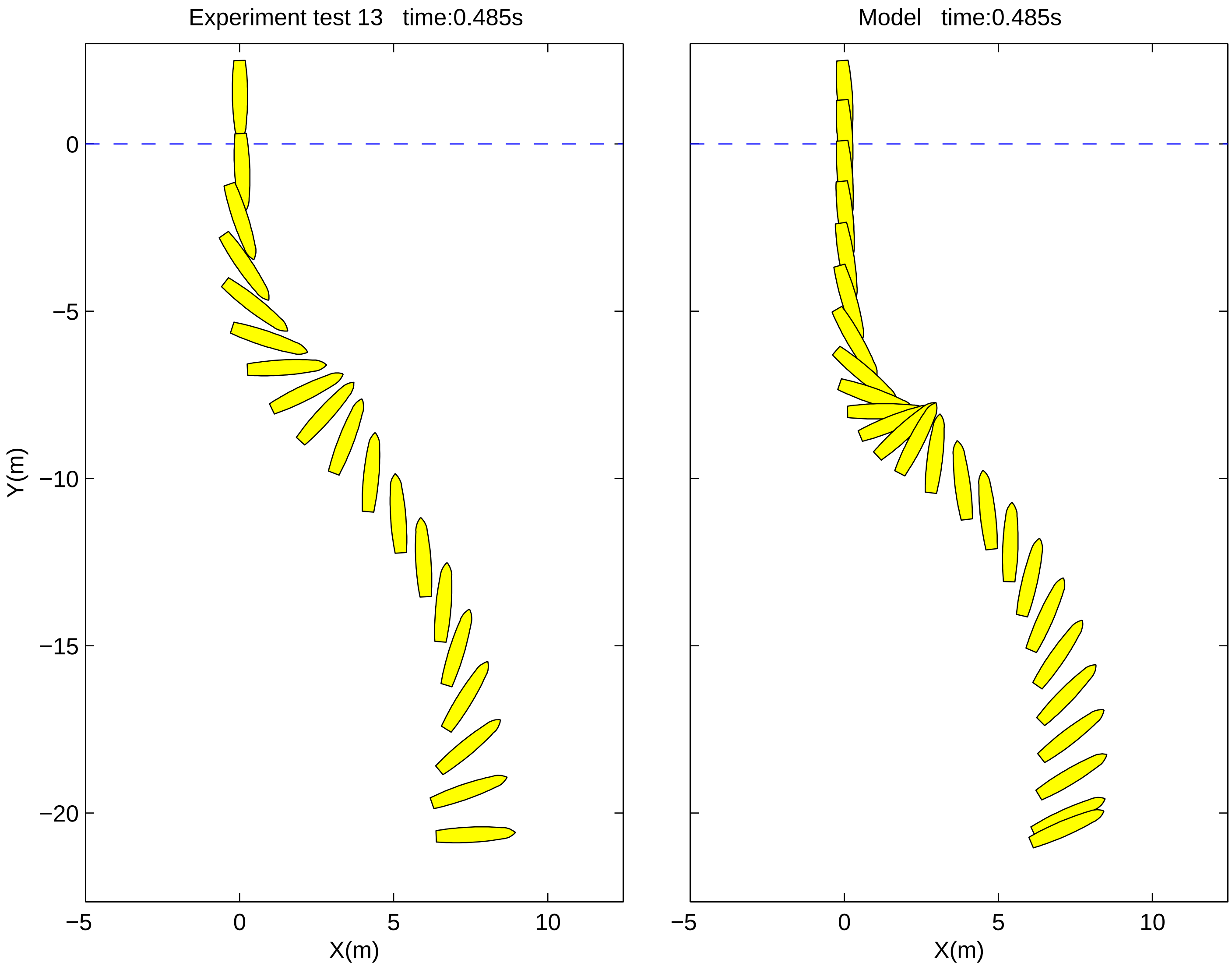
- Test-14
- C_d, C_l, σ_f^*



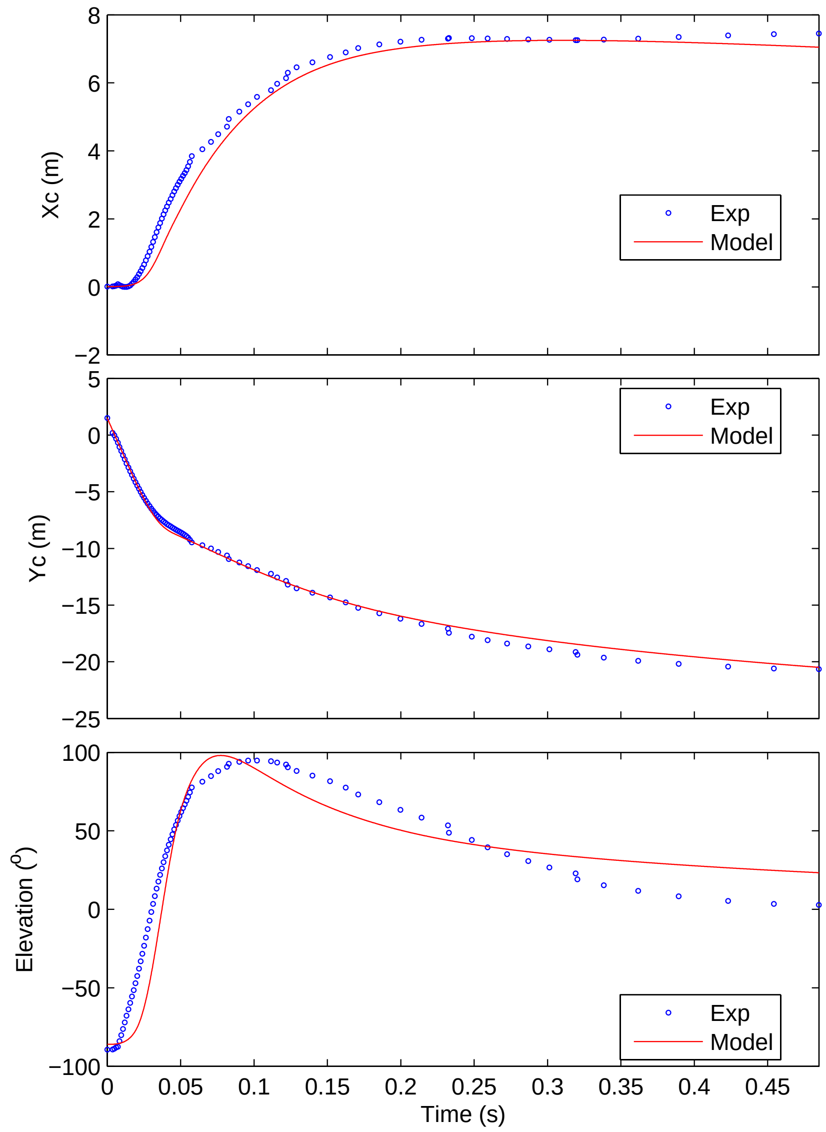
- Test-15
- C_d, C_l, σ_f^*



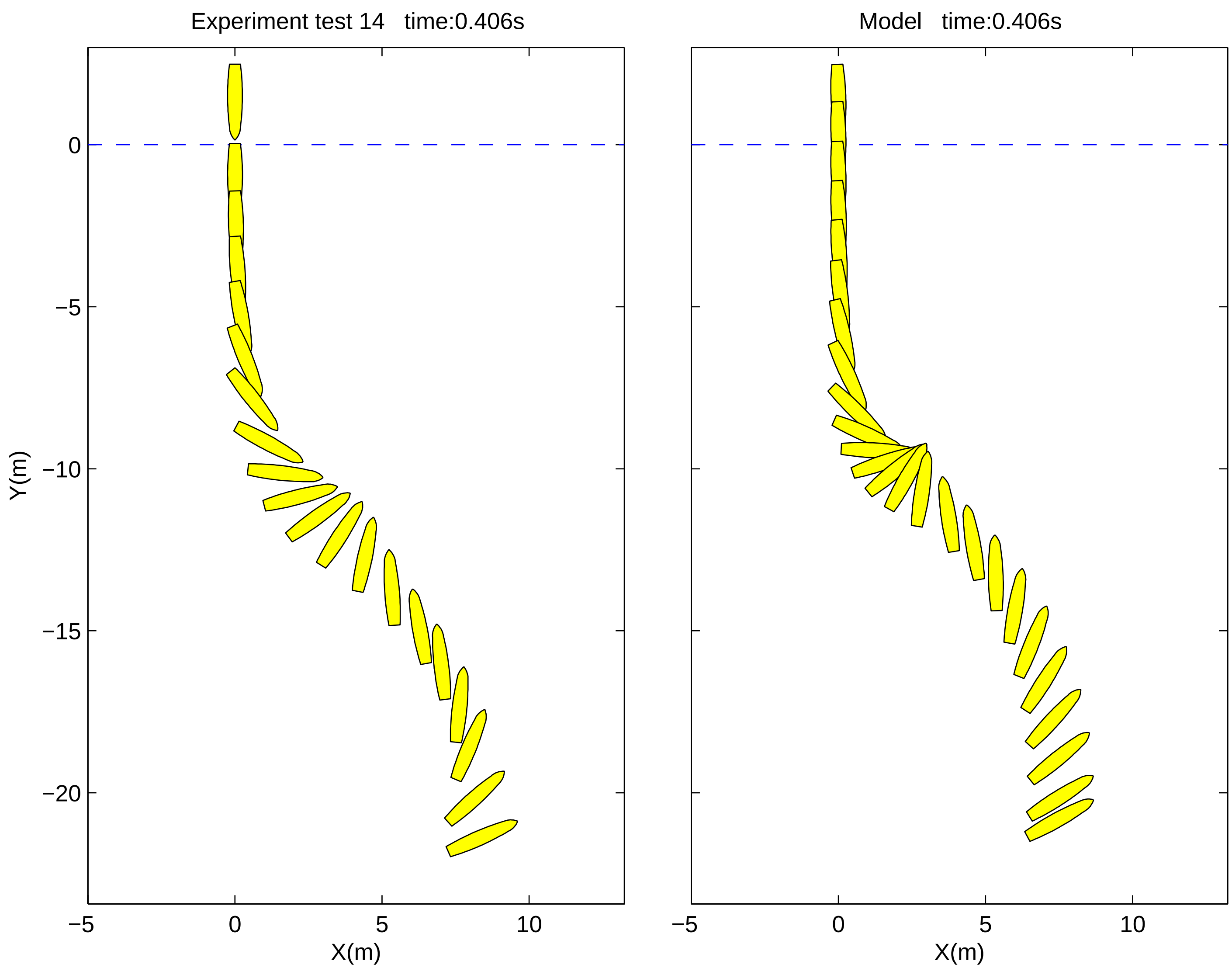
STRIKE35 and SRI Data Inter-Comparison Test-13



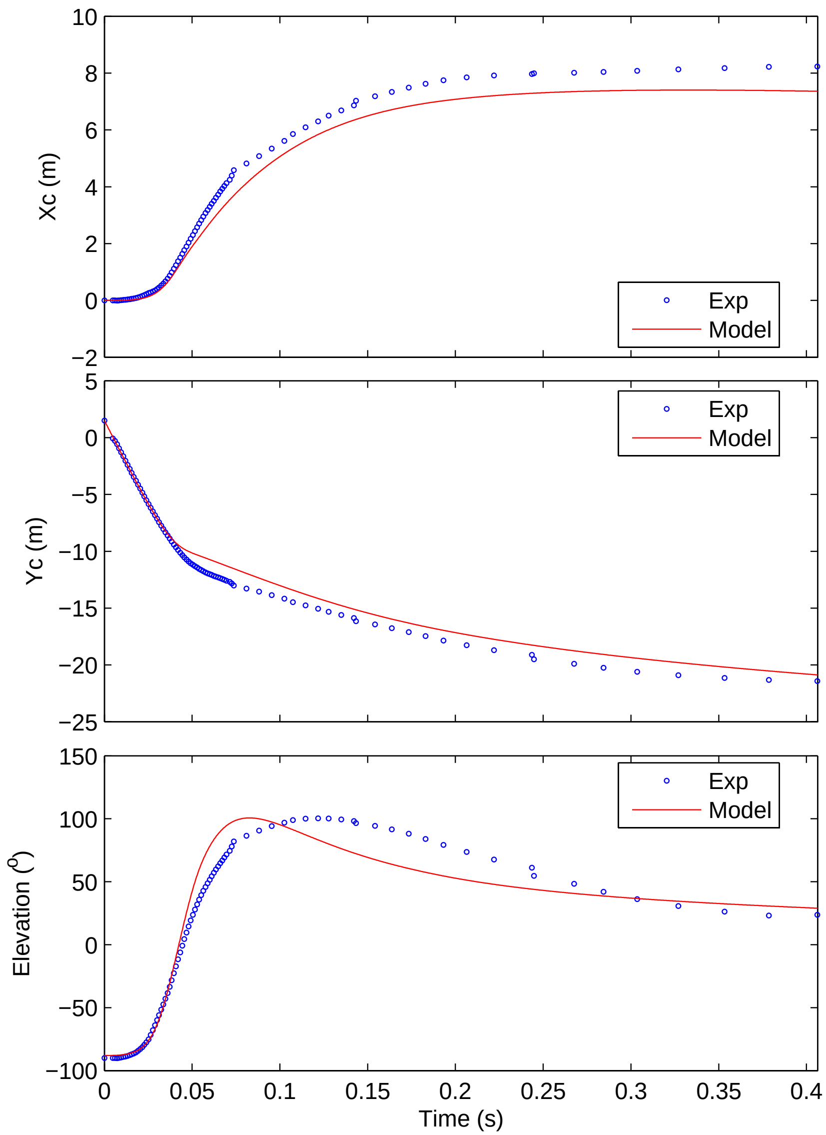
STRIKE35 and SRI Data Inter-Comparison Test-13



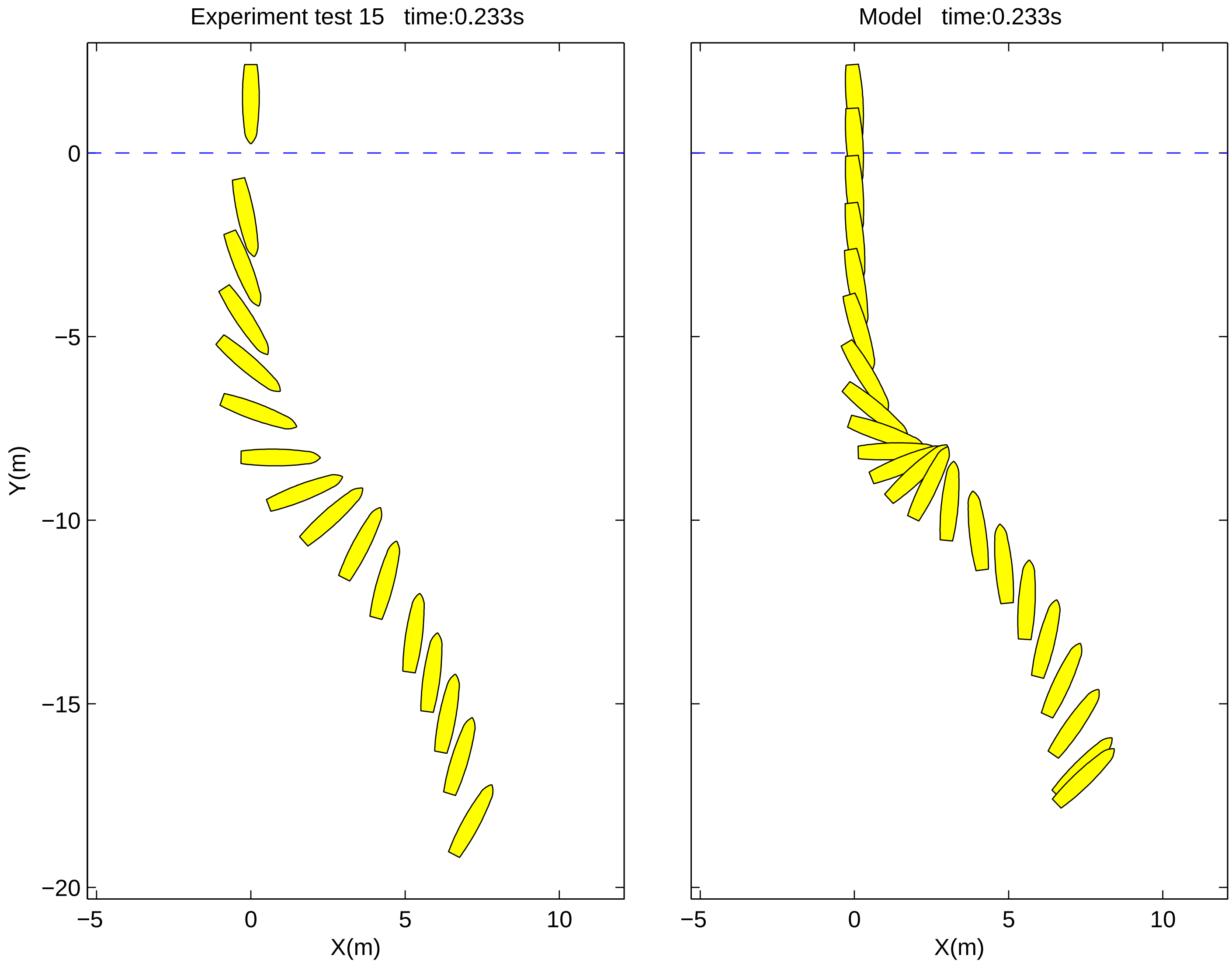
STRIKE35 and SRI Data Inter-Comparison Test-14



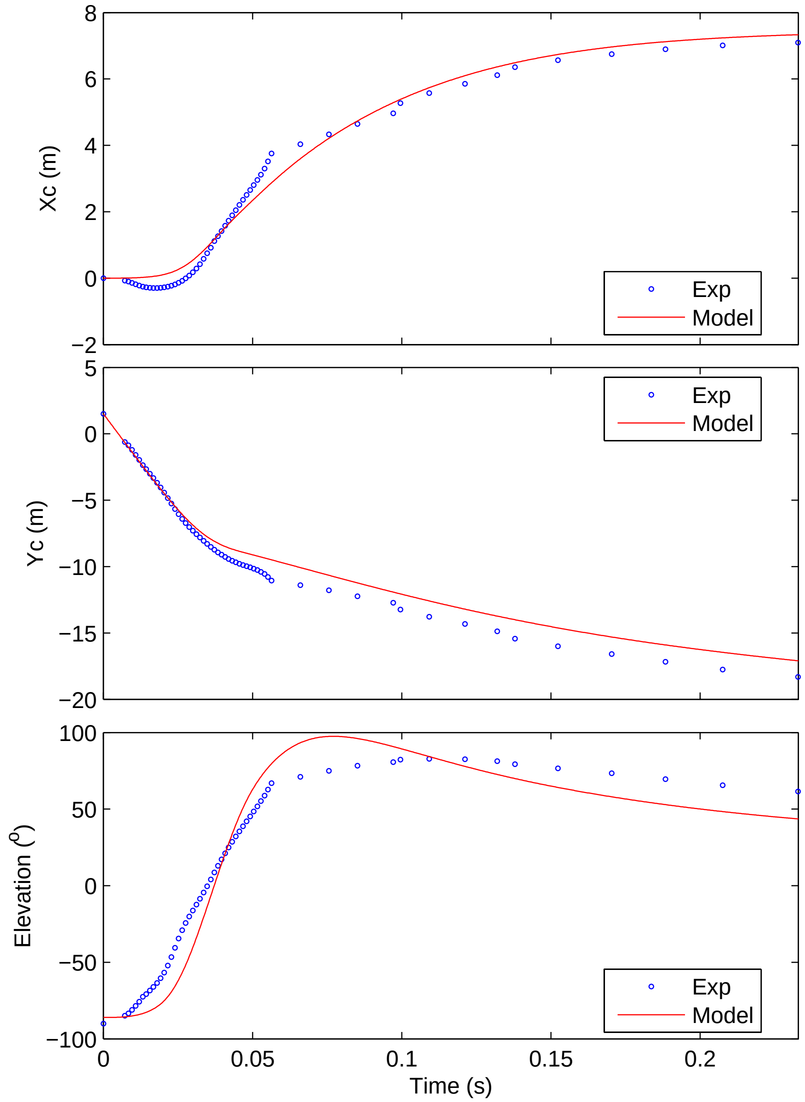
STRIKE35 and SRI Data Inter-Comparison Test-14



STRIKE35 and SRI Data Inter-Comparison
Test-15



STRIKE35 and SRI Data Inter-Comparison Test-15



Conclusions

- (1) STIRKE-35 has capability to predict bomb trajectory.
- (2) A key issue for the prediction is the determination of drag and lift coefficients (C_d, C_l) for a particular bomb.
- (3) Bomb trajectory experiment is needed for determining (C_d, C_l).

LCDR Jillene Bushnell is working on the project for her thesis.